Optical Bremsstrahlung and Transition Radiation from Irradiated Media*

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(Received 11 March 1964)

A semiclassical theory is used to analyze the process by which light is generated when energetic electrons bombard a dielectric slab and undergo multiple scatter in its interior. The electrons are taken to be normally incident from vacuum upon a plane surface bounding the dielectric. Parameters characterizing the material are (a) its complex dielectric permittivity $\epsilon(\omega)$ for light of frequency ω; (b) $\sigma(\Theta)$, the cross section for scatter of an electron through an angle Θ in an encounter with a scattering center in the dielectric; and (c) N, the density of scattering centers, which are taken to be randomly distributed throughout the medium. Light generated due to electric polarization induced in the medium (transition radiation) is considered together with that due to acceleration of the electron in scattering collisions. The distribution function of emitted light describes transition radiation, bremsstrahlung from multiple small-angle electron scattering, and interference between these two kinds of radiation.

I. INTRODUCTION

FOR many years optical experiments have been extremely valuable in investigations of the solid state. Recently, much attention has been given to the processes by which light is generated when electrons interact with matter. Ferrell[1] has suggested that plasmon deexcitation in the neighborhood of a metal surface may result in the emission of light and that this phenomenon may serve as a useful tool for studying the dynamics of electron excitations in metals. His theory of this process was subsequently related[2,3] to the earlier and more general transition radiation theory of Frank and Ginzburg[4] and other workers.[5] In particular, it is shown that the plasmon re-radiation process is completely described by transition radiation theory.

Careful experimental work by Birkhoff, Arakawa, and co-workers[6–8] has confirmed in a very complete way the validity of transition radiation theory. This theory assumes that a fast charged particle interacts only with electrons in the solid upon which it is incident, resulting in a time- and space-varying polarization which in turn gives rise to dipole radiation at large distances from the system.[9] Further, the charged particle is assumed to suffer negligible energy loss and negligible angular deviation in this interaction. Although this picture is valid when a fast particle penetrates very thin layers of dielectric, it may be in error when the medium is not thin. In this case acceleration of the charged particle itself in the field of ion cores in the dielectric may give rise to optical bremsstrahlung in quantity comparable with, or larger than, the number of photons generated in the transition radiation process. In the region of optical frequencies, one must account for the effect of the medium upon photons so generated. These photons may be refracted by the medium or absorbed in its interior; coherence effects between radiation generated in successive scatterings and between transition radiation and bremsstrahlung must also be taken into account.

We have carried out a semiclassical theoretical treatment of the emission of both optical bremsstrahlung and transition radiation from an electron irradiated dielectric slab. We suggest that observation of such radiations, together with information on the optical constants of the solid, may be used to deduce the mean-square ion core scattering angle per unit length for such electrons in the medium. This quantity is not easy to obtain from direct electron scattering experiments, especially at low and intermediate electron energies, and is of importance since it may be used to obtain information on the effective electron-ion interaction potential in the dielectric. This method also may be of value in plasma diagnostics.

II. THEORY OF THE ELECTROMAGNETIC FIELDS

Since attention is focused on photons in the optical range having energies much less than that of the electron, for the present purposes one may suppose the electron to be a point charge having a well-defined path characterized by instantaneous velocity $\mathbf{v}(t)$ and position

$$\mathbf{r}(t)=\int_{-\infty}^{t}\mathbf{v}(t')dt'$$

at time t. The current $\mathbf{j}(\mathbf{r},t)$ generated by this motion may then be written

$$\mathbf{j}(\mathbf{r},t)=-e\mathbf{v}(t)\delta\left(\mathbf{r}-\int_{-\infty}^{t}\mathbf{v}(t')dt'\right). \tag{1}$$

* A preliminary version of this material was presented at the meeting of the Southeastern Section of The American Physical Society in Knoxville, Tennessee, 4–6 April 1963.

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9 This viewpoint is explored in detail by J. C. Ashley and R. H. Ritchie (to be published).

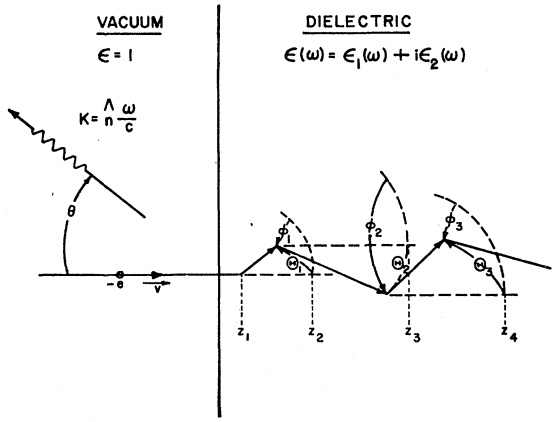

FIG. 1. Geometry of transition radiation and bremsstrahlung generation in multiple scatter of an electron of initial velocity $\mathbf{v}$ in the direction of the $+z$ axis in a dielectric medium. A particular photon is characterized by propagation vector $\mathbf{K}=\hat{n}\omega/c$, where $\hat{n}$ is a unit vector in the direction of emission. The angle between $\hat{n}$ and the $-z$ axis is designated by θ.

It will be assumed that $|\mathbf{v}(t)|\equiv v$ is a constant through all parts of the particle trajectory contributing to the emission of light from the medium, so that one may set $\mathbf{v}(t)=v\hat{u}(t)$, where $\hat{u}$ is a vector of unit length. The Fourier components of $\mathbf{j}$ are

$$\mathbf{j}(k_x,k_y,\omega|z) =\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\int_{-\infty}^{\infty}dte^{i(\omega t-xk_x-yk_y)}\mathbf{j}(x,y,z,t)\,, \quad (2)$$

where the z coordinate axis is chosen normal to the bounding surface and the medium is taken to be infinite in the x and y directions and located in the region $0<z<z_0$ (see Fig. 1). The thickness z_0 is understood to be large compared with the attenuation length of the light observed from the electron's passage through the dielectric but small enough that the distribution function for electron scattering in the dielectric is still strongly peaked in the forward $(+z)$ direction. This condition will be stated quantitatively below in Sec. IV. We put $\hat{u}(t)=\hat{x}\Theta_x(t)+\hat{y}\Theta_y(t)+\hat{z}[1-\frac{1}{2}\Theta^2(t)]$, where $|\mathbf{\Theta}(t)|=\Theta(t)$ is the polar angle which the particle direction makes with the z axis at time t, $\mathbf{\Theta}=\hat{x}\Theta_x+\hat{y}\Theta_y$, and $\hat{x}$, $\hat{y}$, $\hat{z}$ are unit vectors in the direction of the coordinate axes. This form for $\hat{u}(t)$ results from the assumption that $|\mathbf{\Theta}(t)|\ll 1$ for electron trajectories contributing to the emission of light. Requiring that $|\mathbf{j}|^2$ shall contain powers of Θ no higher than the second, one has $j_x(\mathbf{k},\omega|z)=-e\Theta_x(z)e^{i\omega z/v}$, $j_y(\mathbf{k},\omega|z)=-e\Theta_y e^{i\omega z/v}$, and $j_z(\mathbf{k},\omega|z)=-e[1-\tau(z)]e^{i\omega z/v}$, where

$$\tau(z)=\frac{1}{2}\left\{\Theta^2(z)-\frac{i\omega}{v}\int_{-\infty}^{z}\Theta^2(z')dz' +2i\int_{-\infty}^{z}\mathbf{k}\cdot\mathbf{\Theta}(z')dz'+\left[\int_{-\infty}^{z}\mathbf{k}\cdot\mathbf{\Theta}(z')dz'\right]^2\right\}$$

and $\mathbf{k}=\hat{x}k_x+\hat{y}k_y$. From the assumption that the electron traces out a prescribed path, the quantity Θ may be written either as a function of t, or alternatively, as a function of z.

These current components may be used as sources in Maxwell's equations, which are to be solved in the given composite medium. As usual, continuity of tangential electric and magnetic fields across the plane $z=0$ is taken, corresponding to the assumption that $\epsilon(\omega)$ changes discontinuously across the boundary. This solution may be found by standard methods.[3,4] The fields at large distances from the solid are obtained from their Fourier expansions in k_x, k_y space by the saddlepoint method, and from the Poynting flux calculated from these asymptotic fields, the number distribution of photons may be inferred. This distribution divides naturally into two parts: (a) that due to photons polarized perpendicular to the plane of emission (that plane containing the z axis and the direction of observation), denoted by the symbol $\perp$, and (b) that part due to photons polarized parallel to this plane, denoted by the symbol $\|$. One finds that $d^2N_\perp/d\omega d\Omega$, the number of $(\perp)$ photons with angular frequency ω per unit frequency interval emitted in a direction making an angle θ with the negative z axis per unit solid angle around that direction, is given by

$$\frac{d^2N_\perp}{d\omega d\Omega}=\frac{e^2\omega\mu^2}{\pi^2\hbar c^3|\mu+\sigma|^2}\left|\int_0^\infty e^{-\zeta z}[\mathbf{\Theta}(z)\cdot\hat{e}_\perp]dz\right|^2 \quad (3)$$

for the particular scattering history $\mathbf{\Theta}(z)$. In this equation $\hat{e}_\perp$ is a unit vector perpendicular to the plane of emission, $\mu=\cos\theta$, $\sigma=[\epsilon-1+\mu^2]^{1/2}$, $\zeta=-i\omega(1+\beta\sigma)/\beta c$, and $\beta=v/c$.

The distribution of $(\|)$ photons is given by

$$\frac{d^2N_{\|}}{d\omega d\Omega}=\frac{e^2\omega\mu^2}{\pi^2\hbar c^3}\left|\frac{i\beta c}{\omega}\frac{(1-\mu^2)^{1/2}}{(\mu\epsilon+\sigma)}\times\left[\frac{1}{1+\beta\sigma}+\frac{\beta\sigma-\epsilon}{1-\mu^2\beta^2}\right]+\int_0^\infty e^{-\zeta z}g(z)dz\right|^2, \quad (4)$$

where

$$g(z)=\frac{1}{\mu+\sigma}\left\{\mu+\frac{(\epsilon-1)(1-\mu^2)}{\mu\epsilon+\sigma}\right\}(\mathbf{\Theta}(z)\cdot\hat{e}_{\|})-\frac{(1-\mu^2)^{1/2}}{\mu\epsilon+\sigma}\lambda(z)\,,$$

and

$$\lambda(z)=\frac{1}{2}\left\{\Theta^2(z)-\frac{i\omega}{\beta c}\int_0^z\Theta^2(z')dz'+2\frac{i\omega}{c}\int_0^z\mathbf{\Theta}(z')\cdot\hat{e}_{\|}dz' +\frac{\omega^2(1-\mu^2)}{c^2}\left[\int_0^z\mathbf{\Theta}(z)\cdot\hat{e}_{\|}dz'\right]^2\right\}.$$

The unit vector $\hat{e}_{\|}$ is parallel to the plane of emission and perpendicular to $\hat{z}$.

III. ENSEMBLE AVERAGE

These expressions must now be averaged over statistically independent scattering histories. The electron trajectories are taken to consist of straight line segments between scatterings and discontinuous changes in direction at scattering points. A given history is characterized by the infinite sequence z_1, $\mathbf{\Theta}_1$; z_2, $\mathbf{\Theta}_2$, z_3, $\mathbf{\Theta}_3$; $\cdots z_1$, $\mathbf{\Theta}_i$; $\cdots$, where z_i is the depth in the solid at which the ith scatter occurs, and $\mathbf{\Theta}_i$ describes the direction of the electron relative to $\hat{z}$ after the ith scatter. The probability that this sequence shall occur may be written, again in the small angle approximation,

$$\mathbf{W}d\tau = e^{-\gamma z_1}\gamma dz_1 p(|\mathbf{\Theta}_1|)d\Omega_1 e^{-\gamma(z_2-z_1)}\gamma dz_2 p(|\mathbf{\Theta}_2-\mathbf{\Theta}_1|)d\Omega_2 \times e^{-\gamma(z_3-z_2)}\gamma dz_3 p(|\mathbf{\Theta}_3-\mathbf{\Theta}_2|)d\Omega_3\cdots, \quad (5)$$

where $p(\Theta)d\Omega=\sigma(\Theta)d\Omega/\int\sigma(\Theta)d\Omega$ is the probability that, in a single scattering event, the electron is deflected elastically through a polar angle Θ into the infinitesimal solid angle $d\Omega$, $\gamma=N\int\sigma(\Theta)d\Omega$ is the inverse mean free path for scatter in the solid, $\sigma(\Theta)$ is the differential scattering cross section per scattering center, and N is the average number of scattering centers per unit volume in the solid.

It is not necessary here to attribute angular deviation solely to the process of elastic scatter on ion cores. Elastic scattering on electrons in the medium can be included in a formal sense by redefining the cross section $\sigma(\Theta)$. Since our final results [Eqs. (7) and (8)] contain only the mean-square scattering angle of electrons in the dielectric, it may be understood to contain contributions from all processes, including angular deviation from inelastic scattering as long as the fractional change in the electron energy is small. However, it is well known that elastic scatter on ion cores gives rise to substantially large angular deviations than the other processes in materials having atomic numbers appreciably greater than unity. This follows from the dependence of mean-square elastic scattering angle on the square of the effective atomic number of the medium [see Eq. (9) below].

The averaged distribution function

$$\langle d^2N_\perp/d\omega d\Omega\rangle = \int \mathbf{W}d\tau\{d^2N_\perp/d\omega d\Omega\} \quad (6)$$

may be evaluated by elementary methods (see Appendix A). One finds

$$\langle d^2N_\perp/d\omega d\Omega\rangle = \frac{\alpha\beta^2\mu^2\langle\Theta^2\rangle_l}{2\pi^2\omega|1+\beta\sigma|^2|\mu+\sigma|^2\left(2\,\mathrm{Re}\left\{-\dfrac{i\omega}{c}\sigma\right\}\right)}, \quad (7)$$

where $\langle\Theta^2\rangle_l=N\int\Theta^2\sigma(\Theta)d\Omega$ is the mean-square scattering angle per unit length in the medium, and $\alpha=e^2/\hbar c$. The factors in this expression have straightforward interpretations: $|\mu+\sigma|^{-2}$ contains the effect of reflection of photons at the surface of the solid, $|1+\beta\sigma|^{-2}$ is a relativistic factor which describes peaking of the radiation pattern in the $\hat{z}$ direction at high velocities, and $\{2\,\mathrm{Re}(-i\omega\sigma/c)\}^{-1}$ is the e-folding distance for absorption in the solid of photons of frequency ω emerging from the dielectric at angle θ.

This simple result is exact and, of course, does not depend upon the particular scattering law, as long as small angle scattering obtains.

The corresponding result for the averaged distribution of photons polarized parallel with the plane of emission is

$$\left\langle\frac{d^2N_{\parallel}}{d\omega d\Omega}\right\rangle = \frac{\alpha\beta^2\mu^2}{\pi^2\omega}\left\{\frac{(1-\mu^2)}{|\mu\epsilon+\sigma|^2}\left|\frac{1}{1+\beta\sigma}+\frac{\beta\sigma-\epsilon}{1-\beta^2\mu^2}\right|^2+\left[\frac{1}{|1+\beta\sigma|^2}\left|\frac{\mu}{\mu+\sigma}+\frac{1-\mu^2}{\mu\epsilon+\sigma}\left(\frac{\epsilon-1}{\mu+\sigma}+\frac{\beta}{1+\beta\sigma}\right)\right|^2\frac{1}{4\,\mathrm{Re}(\zeta)}\right.\right.$$
$$\left.\left.-\frac{1-\mu^2}{|\mu\epsilon+\sigma|^2}\mathrm{Re}\left(\frac{1}{\zeta(1+\beta\sigma)^2}\left\{2+\beta\sigma-\frac{\beta^2(1-\mu^2)}{1+\beta\sigma}\right\}\left\{\frac{1}{1+\beta\sigma^*}+\frac{\beta\sigma^*-\epsilon^*}{1-\beta^2\mu^2}\right\}\right)\right]\langle\Theta^2\rangle_l\right\}, \quad (8)$$

where terms containing powers of Θ no higher than the second have been included. The first term in the curly brackets arises from polarization in the medium (transition radiation) while the first term in the square brackets may be thought of as arising from scattering of the incident electron on nuclei (bremsstrahlung). The last term in Eq. (3) represents interference between transition radiation and bremsstrahlung. Such interference does not arise in the case of perpendicularly polarized photons, since transition radiation is polarized entirely in the parallel direction.

To compare these results with those appropriate to single scattering *in vacuo*, one may let $N\to 0$ and $\mathrm{Re}\{\epsilon\}\to 1$ in Eqs. (7) and (8), so that $\langle\Theta^2\rangle_l\lambda_{\mathrm{opt}}\to 0$, where $\lambda_{\mathrm{opt}}\equiv\{2\,\mathrm{Re}(-i\omega\sigma/c)\}^{-1}$ is the optical attenuation length. These formulas then may be looked at as representing radiation due, in this limit, to single scattering on very dilute centers uniformly distributed in a purely absorbing medium, the radiation from which is attenuated a factor of e in a distance λ_{opt}. The single scattering limit is thus obtained by dividing both of these equations by the factor $N\lambda_{\mathrm{opt}}$, and then setting $\epsilon=1$. One finds

$$\left\langle\frac{d^2N_\perp}{d\omega d\Omega}\right\rangle\rightarrow\frac{\alpha\beta^2}{8\pi^2\omega(1+\beta\mu)^2}\frac{\langle\Theta^2\rangle_l}{N}\equiv\left(\frac{d^2\sigma_\perp}{d\omega d\Omega}\right)_{\rm ss} \quad (7a)$$

$$\left\langle\frac{d^2N_{\parallel}}{d\omega d\Omega}\right\rangle\rightarrow\left(\frac{\mu+\beta}{1+\beta\mu}\right)^2\left(\frac{d^2\sigma_\perp}{d\omega d\Omega}\right)_{\rm ss}. \quad (8a)$$

In order to compare the predictions of Eqs. (7a) and (8a) with independent results on single scattering bremsstrahlung yields in the backward hemisphere of directions, one may consider the case of elastic scattering of high-energy electrons on a screened nuclear charge. From the work of Nigam, Sundaresan, and Wu,[10] the mean-square scattering angle for electrons interacting with a nucleus of charge Ze *via* an exponentially screened Coulomb field may be written

$$\frac{\langle\Theta^2\rangle_l}{N}=\frac{8\pi r_0^2Z(Z+1)(1-\beta^2)}{\beta^4}\times\left[\ln\frac{137\beta}{Z^{1/3}(1-\beta^2)^{1/2}}+\ln\frac{1.76}{\nu}-(1+\beta^2/4)\right] \quad (9)$$

in the first Born approximation. In this equation ν is $O(1)$. This result was obtained from their Eq. (56a) for Q_l, where one takes the quantity $\langle\Theta^2\rangle_l/N$ to be approximately equal to $2Q_2/3Nt$ in the notation of Ref. 10. Using Eq. (9) above in Eqs. (7a) and (8a), one finds

$$\left(\frac{d^2\sigma_\perp}{d\omega d\Omega}\right)_{\rm ss}=\frac{\alpha\beta^2(1-\beta^2)Z(Z+1)r_0^2}{\pi\beta(1+\beta\mu)^2}\times\left\{\ln\frac{137\beta}{Z^{1/3}(1-\beta^2)^{1/2}}+O(1)\right\} \quad (10)$$

$$\left(\frac{d^2\sigma_{\parallel}}{d\omega d\Omega}\right)_{\rm ss}=\left(\frac{\mu+\beta}{1+\beta\mu}\right)^2\left(\frac{d^2\sigma_\perp}{d\omega d\Omega}\right)_{\rm ss}, \quad (11)$$

where $r_0=e^2/mc^2$ and where a screening factor appropriate to the Thomas-Fermi atom model has been used. These formulas differ only in minor particulars from the results of Gluckstern and Hull[11] on the low-energy bremsstrahlung due to single scatter of relativistic electrons on a screened charge center. Their Eqs. (11.3) and (11.2), in the limit $\hbar\omega\ll mc^2$, correspond to Eqs. (10) and (11) above, respectively, except for terms $O(1)$ in the curly brackets and in the use of Z^2 instead of $Z(Z+1)$ since they did not consider bremsstrahlung due to scatter on atomic electrons. Recall that the quantity μ in the present usage is equivalent to $(-\cos\theta_0)$ in the notation of Ref. 11. Thus one sees that Eqs. (7) and (8) reduce to the results of Gluckstern and Hull[11] for low-energy bremsstrahlung generation by a high-energy electron scattering on a screened nucleus of charge Ze in the limiting case of single scatter on an isolated nucleus. This comparison is made by using an equation for $\langle\Theta^2\rangle_l$ appropriate to such scattering in Eqs. (7) and (8).

[10] B. P. Nigam, M. K. Sundaresan, and E. T. Wu, Phys. Rev. **115**, 491 (1959).

[11] R. L. Gluckstern and M. H. Hull, Jr., Phys. Rev. **90**, 1030 (1953).

IV. RANGE OF VALIDITY OF THE PRESENT TREATMENT

The considerations of this paper apply to a dielectric slab which is thick compared with the optical attenuation length ($z_0\gg\lambda_{\rm opt}$) but is thin enough that backscattered electrons may be neglected ($\langle\Theta^2\rangle_l z_0\ll1$).

It is clear that the treatment is also restricted to photon energies low enough that photon transport may be described adequately by a dielectric approach, i.e., multiple photon scatter is neglected.

Since energy loss by the incident particle is not considered, one must also have $-(dW(E)/dS)(\lambda_{\rm opt}/E)\ll1$, where $-dW(E)/dS$ is the energy loss per cm of path length in the dielectric by an electron of energy E.

V. CONCLUSIONS

A simple result has been obtained for the light emitted from a dielectric medium irradiated normally by energetic electrons which may polarize the medium and which may undergo multiple scattering in the medium. The medium must not be so thick that backscattered electrons contribute to the emission of photons at the entrant surface.

We propose that $\langle\Theta^2\rangle_l$ may be determined from measurements of the distribution of polarized photons emitted from an electron irradiated solid, if $\epsilon(\omega)$ is known for that solid, by employing either Eq. (7) or (8) above. By observing photons which are strongly attenuated in the solid, one effectively confines attention to scatterings which occur within a small distance of the surface. This depth may be small enough so that one perceives the effect of electron scattering only while the electron distribution function is strongly peaked in the forward direction and well before the "diffusion" stage is reached.[12]

This method has the advantage that it may be used to determine $\langle\Theta^2\rangle_l$ inside a solid material at electron energies so low that the electron distribution function itself would be very difficult to measure.

APPENDIX A. EVALUATION OF STATISTICAL AVERAGES

After the integration over z in Eq. (3) has been carried out, the average in Eq. (6) may be written

$$\langle d^2N_\perp/d\omega d\Omega\rangle=J=C\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\int \mathbf{W}d\tau S_mS_n^*T_mT_n \quad (A1)$$

[12] H. A. Bethe, M. E. Rose, and L. P. Smith, Proc. Am. Phil. Soc. **78**, 573 (1938).

where

$$T_m = \mathbf{\Theta}_m \cdot \hat{e}_\perp ,$$

$$S_m = \frac{1}{\zeta}(e^{-\zeta z_m} - e^{-\zeta z_{m+1}}) ,$$

$$C = e^2 \omega \mu^2 / (\pi^2 \hbar c^3 |\mu + \sigma|^2) ,$$

and the asterisk denotes complex conjugation.

The average of $T_m T_n$ over the angular distributions may be shown to give $\frac{1}{2}(g_{mn})\langle\Theta^2\rangle_{\rm av}$, where $\langle\Theta^2\rangle_{\rm av} = \int d\Omega p(\Theta)\Theta^2$, and where $g_{mn} = \min(m,n)$. It was assumed in evaluating this average that the elastic scattering cross section is peaked strongly in the forward direction and hence that $\mathbf{\Theta}$ may be considered a two-dimensional vector perpendicular to $\hat{z}$.

The integrations over the z variables may be carried out explicitly and yield

$$J = \frac{C\langle\Theta^2\rangle_{\rm av}}{2|\zeta|^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} g_{mn} \times \{ I_{mn} + I_{m+1,n+1} - I_{m+1,n} - I_{m,n+1} \} , \quad \text{(A2)}$$

where

$$I_{mn} = \left(\frac{\gamma}{\gamma + 2\,\mathrm{Re}\zeta} \right)^{g_{mn}} \times \left(\frac{\gamma}{\gamma + \mathrm{Re}\zeta + i\,\mathrm{sgn}(m-n)\,\mathrm{Im}(\zeta)} \right)^{|m-n|} . \quad \text{(A3)}$$

The function $\mathrm{sgn}(x) \equiv x/|x|$. The double sum may be evaluated without difficulty; the final result is

$$J = C\gamma\langle\Theta^2\rangle_{\rm av}/4|\zeta|^2\,\mathrm{Re}\zeta . \quad \text{(A4)}$$

It is clear that $\langle\Theta^2\rangle_l = \gamma\langle\Theta^2\rangle_{\rm av}$. Inserting the factor C into Eq. (A4), one finds Eq. (7). The averages over $d^2N_{11}/d\omega d\Omega$ may be evaluated in the same manner as that sketched above.

PHYSICAL REVIEW VOLUME 135, NUMBER 3A 3 AUGUST 1964

Magneto-Optical Studies of the Band Structure of PbS

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By a combination of interband and free-carrier experiments performed on epitaxial and bulk n- and p-type crystals, the energy-band parameters of PbS in the vicinity of the band edge have been determined at 77°K. Interband magneto-absorption measurements have established that the valence and conduction bands are nondegenerate (except for spin), approximately spherical and parabolic, and have extrema at essentially the same point in $\mathbf{k}$ space, probably the L points. Analysis of the data yields values for the energy gap, the reduced effective mass and also the effective g factors for both the valence and conduction bands. Measurements of the Burstein-Moss effect in the zero-field absorption edge indicate multiple bands, most likely at the <111> zone boundaries. The individual valence- and conduction-band effective masses were determined from free-carrier Faraday rotation measurements at a temperature slightly above 77°K. Additional structure in the rotation for p-type material indicates possible intervalence band transitions. The reversal of the interband Faraday rotation with carrier concentration and temperature is explained on the basis of the band structure determined here. The effects of strain and carrier concentration on the band parameters were also investigated and the effective deformation potential for dilation was determined.

I. INTRODUCTION

MAGNETO-OPTICAL experiments[1] provide a powerful tool for probing the band structure of semiconductors in the vicinity of band extrema. Cyclotron resonance[2] is particularly useful since it yields precise values for effective masses and their anisotropy. The free-carrier Faraday[3] and Voigt[4] effects, which arise from the dispersion associated with the cyclotron absorption lead to less detailed information, but are useful since they may be measured over a wider range of experimental conditions. The free-carrier experiments, however, suffer a limitation in that they only

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